# “Collisionless” Complex Reconnection

$$\mathbf{B} \simeq B_0 \mathbf{e}_z + B_y(x) \mathbf{e}_y$$

$$\hat{\mathbf{E}} \simeq -\nabla \hat{\Phi} - \mathbf{e}_z \frac{1}{c} \frac{\partial \hat{A}_z}{\partial t}$$

$$\hat{\Phi} = \tilde{\phi}(x) \exp\left(-i\omega t + ik_y y + ik_z z\right)$$

$$\mathbf{k} \cdot \mathbf{B}(x = x_0) = 0 \qquad \mathbf{k} \cdot \mathbf{B} \simeq k_y B_y' (x - x_0)$$

**"Translated" into toroidal geometry where the diamagnetic velocities are in the TOROIDAL DIRECTION**

# Inductive Mode

$$0 \simeq -\nabla_{\parallel}\hat{p}_e - \frac{\widehat{B}_x}{B}\frac{dp_e}{dx} - en\left[\widehat{E}_{\parallel} + i\omega\mathcal{L}\hat{J}_{\parallel}\right]$$

$$\frac{1}{r_J} = \frac{1}{\widetilde{B}_{x0}}\left\{\left.\frac{d\widetilde{B}_x}{dx}\right|_{x=x_{0+}} - \left.\frac{d\widetilde{B}_x}{dx}\right|_{x=x_{0-}}\right\}$$

$$S_L \equiv \frac{c^2}{4\pi}\mathcal{L}$$

$$\delta\overline{\omega} \simeq \left(1 - \frac{\omega_{*e}^p}{\omega_{di}}\right)\frac{\omega_{di}^2\omega_H^2}{\left(\omega_{di} - \omega_{*e}^p\right)^4}\frac{k^2 S_L^3}{\left(r_J I\right)^4} \sim \frac{\omega_H^2}{\omega_{di}^2}\frac{k^2 S_L^3}{r_J^4}$$

$$\omega \simeq (1 - \delta\bar{\omega})\,\omega_{di} + i\gamma,$$

$$\gamma = \nu_{\mu}\delta\bar{\omega}.$$

$$\delta \simeq \frac{S_{L}}{r_{J} I} \frac{dp_i/dx}{d\,(p_i + p_e)\,/dx}.$$

# “Naive” Resistive Mode

$$0 \simeq -\nabla_{\|}\hat{p}_e - \frac{\hat{B}_x}{B}\frac{dp_e}{dx} - en\left(\hat{E}_{\|} - \eta_{\|}\hat{J}_{\|}\right) - \alpha_t n \nabla_{\|}\hat{T}_e$$

$$D_m \equiv \eta_{\|}\frac{c^2}{4\pi}$$

$$\gamma \simeq \frac{(\omega_{di} - \omega_{*e})\,\omega_{\mathcal{H}}^2}{\omega_{di}\,(\omega_{di} - \omega_{*e}^p)^4}\frac{k^2}{(r_J I)^4}D_m^3 > 0.$$

$$\delta^4 \simeq \left[\frac{D_m}{r_J I\,|\omega_{di} - \omega_{*e}^p|}\right]^4.$$

$$D_m > D_B^i \frac{\rho_i}{r_{pi}}\,(k_\perp r_J I)$$

## Combined Transport Reconnecting Mode

Finite thermal conductivity, resistivity and ion momentum diffusion play an important role

## Longitudinal Electron Momentum Conservation Equation

$$\tilde{B}_x \simeq i(\mathbf{k}\cdot\mathbf{B})\tilde{\xi}_x + S_m \frac{d^2\tilde{B}_x}{dx^2}$$

where $\tilde{\xi}_x$ represents the radial displacement,

$$S_m \equiv D_m \frac{1 + i\dfrac{\hat{D}_*}{\bar{\varpi}}\bar{x}^2\left[1 + \dfrac{2}{3}(1+\alpha_T)^2 \Big/ \left(1 + i\dfrac{\hat{D}_\parallel}{\bar{\varpi}}\bar{x}^2\right)\right]}{\omega - \omega_{*e} - \omega_{*e}^T \Big/ \left(1 + i\dfrac{\hat{D}_\parallel}{\bar{\varpi}}\bar{x}^2\right)}$$

where

$$D_m \equiv \eta_{\|} \frac{c^2}{4\pi}, \; \bar{\omega} \equiv \frac{\omega}{\omega_0} \;, \; \bar{x} \equiv (x - x_0)/\delta_0$$

$$\omega_{*e} \equiv -k_y \frac{cT_e}{eBn} \frac{dn}{dx} \qquad \omega_{*e}^{T} \equiv -k_y \frac{c}{eB} \frac{dT_e}{dx}$$

$$\bar{D}_* = \left( k_y \delta_0 \frac{B_y'}{B_0} \right)^2 \frac{2T_e}{m_e \nu_{ei} \omega_0} \;, \quad \bar{D}_{\|} \simeq \left( k_y \delta_0 \frac{B_y'}{B_0} \right)^2 \frac{T_e}{m_e \nu_{ee} \omega_0},$$

$\delta_0$ and $\omega_0$ are normalization quantities, and $\tilde{B}_x \simeq \tilde{B}_{x0} = const$.

Defining

$$Y \equiv ik_y B_y' \delta \frac{\bar{\xi}_x}{\tilde{B}_{x0}}$$

we have

$$\frac{1}{\tilde{B}_{xo}} \frac{d^2 \tilde{B}_x}{dx^2} \simeq \frac{1}{S_m}\left(1 - \bar{x}Y\right)$$

Then

$$\frac{1}{r_s} = \delta_0 \int_{-\infty}^{+\infty} d\bar{x} \frac{1}{\tilde{B}_{x0}} \frac{d^2 \tilde{B}_x}{dx^2} \simeq \delta_0 \int d\bar{x} \frac{1}{S_m}\left(1 - \bar{x}Y\right)$$

The ion momentum conservation equation gives

$$\boxed{\frac{\omega-\omega_{di}}{\omega_H^2}\left(\omega - i\frac{D_\mu}{\delta_0^2}\frac{d^2}{d\bar{x}^2}\right)\frac{d^2Y}{d\bar{x}^2} \simeq \frac{k^2}{S_m}\delta_0^4\bar{x}\left(1-\bar{x}Y\right)}$$

where

$$\omega_H^2 \equiv \frac{B_y'^2}{4\pi m_i n} \quad \text{and} \quad \omega_{di} \equiv k_y\frac{c}{eBn}\frac{dp_i}{dx}$$

# Electron Thermal Energy Balance Equation

Simplest (cylindrical) form

$$\left(-i\omega + D_{\|}^2 k_{\|}^2\right)\hat{T}_e + T_e'\left(\hat{V}_{Er} - ik_{\|}\frac{\hat{B}_r}{B}D_{\|}\right) + ik_{\|}T_e\hat{U}_{e\|}\frac{2}{3}\left(1+\alpha_T\right) \simeq 0$$

That gives

$$\hat{T}_e \simeq \frac{1}{\left(\omega + iD_{\|}k_{\|}^2\right)}\left[\left(\omega\hat{\xi}_r + k_{\|}\frac{\hat{B}_r}{B}D_{\|}\right)\left(-T_e'\right) + \frac{2}{3}k_{\|}\hat{U}_{e\|}T_e\left(1+\alpha_T\right)\right]$$

for $\hat{V}_{Er} = -i\omega\hat{\xi}_r$

However $D_\perp^e$ is anomalous (with $D_\perp^e \sim D_\mu^i$)

Therefore Eq. (1) should be replaced by

$$\left(-i\omega + D_\parallel^e k_\parallel^2\right)\hat{T}_e - D_\perp^e \frac{\partial^2 \hat{T}_e}{\partial x^2}$$

$$\simeq i\left(\omega\hat{\xi}_x + k_\parallel \frac{\hat{B}_x}{B} D_\parallel^e\right)\frac{dT_e}{dx} - ik_\parallel \hat{U}_{e\parallel} T_e \frac{2}{3}\left(1+\alpha_T\right) \quad (2)$$

## Longitudinal Electron Momentum Balance Equation

(considering a realistic anomalous transverse electron thermal conductivity)

$$-en\hat{E}_{\parallel} - \nu_{ei}^{\parallel} m_e n \hat{u}_{e\parallel} - \hat{\Im}_T^e \simeq 0$$

$$\boxed{\hat{\Im}_T^e \equiv T_e \nabla_{\parallel} \hat{n}_e + n\left(1+\alpha_T\right)\nabla_{\parallel}\hat{T}_e + \frac{\hat{B}_{x0}}{B}\left[T_e \frac{dn}{dx} + \left(1+\alpha_T\right) n \frac{dT_e}{dx}\right]}$$

where

$$\hat{n}_e \simeq -n'\hat{\xi}_x + \frac{k_{\parallel}}{\omega}\hat{u}_{e\parallel} n$$

and $\hat{V}_{Ex} = -i\omega\xi_x$

Then

$$-en\hat{E}_{\|} - m_e n\hat{u}_{e\|}\left(v_{ei}^{\|} + i\frac{k_{\|}^2 T_e}{m_e\omega}\right) + ik_{\|}T_e\frac{dn}{dx}\hat{\xi}_x$$

$$-\frac{\tilde{B}_{x0}}{B}\left[T_e\frac{dn}{dx} + (1+\alpha_T)n\frac{dT_e}{dx}\right] - i(1+\alpha_T)nk_{\|}\hat{T}_e \simeq 0$$

We assume the ordering of **maximum information**, considering these estimates.

$$\omega \sim \omega_{di} \sim \frac{k\rho_i v_{ti}}{2r_{pi}} \sim \frac{D_\mu}{\delta_\mu^2} \sim \nu_{ii}\frac{\rho_i^2}{4\delta_\mu^2} \qquad ; \qquad \frac{1}{r_{pi}} \equiv -\frac{1}{p_i}\frac{dp_i}{dx}$$

$$\omega\nu_{ee} \sim k^2\delta_\parallel^2\frac{v_{te}^2}{L_S^2} \qquad ; \qquad \frac{1}{L_S^2} \equiv \left(\frac{B_y'}{B_0}\right)^2$$

$$\delta_\mu^2 \sim \frac{r_{pi}}{2\lambda_i}\frac{\rho_i}{k} \qquad \delta_\parallel^2 \sim \frac{\rho_i}{k}\frac{1}{2r_{pi}}\frac{1}{\lambda_e}\left(\frac{v_{te}}{v_{ti}}\right)^{-1}L_S^2 \qquad ; \qquad \lambda_e \equiv \frac{v_{te}}{\nu_{ee}}$$

Clearly, $\delta_{\|} \sim \delta_{\mu}$ provided that

$$\frac{L_S^2}{4r_{pi}^2}\left(\frac{m_e}{m_i}\right)^{1/2} \sim 1$$

Moreover, we also require that

$$\frac{\omega_H^2 k^2 \rho_i^4}{\omega_{di}\nu_{ei}d_e^2} \sim \frac{k^2}{L_S^2}\rho_i^4 \frac{\Omega_{ce}\Omega_{ci}}{\omega_{di}\nu_{ei}} \sim k\rho_i \frac{\lambda_e}{2r_{pi}}\left(\left(\frac{m_i}{m_e}\right)^{1/2}\frac{4r_{pi}^2}{L_S^2}\right) < 1$$